

Exercise 6

If $f(x) = x - 1$ and $g(x) = 1/(x + 1)$, find the following.

- | | |
|----------------|----------------|
| a. $f(g(1/2))$ | b. $g(f(1/2))$ |
| c. $f(g(x))$ | d. $g(f(x))$ |
| e. $f(f(2))$ | f. $g(g(2))$ |
| g. $f(f(x))$ | h. $g(g(x))$ |

Solution

Evaluate each of the function compositions.

$$f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 = \frac{1 - (x+1)}{x+1} = \frac{-x}{x+1} = -\frac{x}{x+1}$$

$$f\left(g\left(\frac{1}{2}\right)\right) = -\frac{\frac{1}{2}}{\frac{1}{2} + 1} = -\frac{1}{3}$$

$$g(f(x)) = g(x-1) = \frac{1}{(x-1)+1} = \frac{1}{x}$$

$$g\left(f\left(\frac{1}{2}\right)\right) = \frac{1}{\frac{1}{2}} = 2$$

$$f(f(x)) = f(x-1) = (x-1) - 1 = x - 2$$

$$f(f(2)) = 2 - 2 = 0$$

$$g(g(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{1}{x+1} + \frac{x+1}{x+1}} = \frac{1}{\frac{1+(x+1)}{x+1}} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$$

$$g(g(2)) = \frac{2+1}{2+2} = \frac{3}{4}$$